Area Exam Syllabus

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March 10, 2025

1 Wave Equations and General Relativity

(a) Local existence and uniqueness for linear wave equations.

(b) Local existence and uniqueness for nonlinear wave equations.

(c) Dispersive estimates for linear wave equations on whole space using exact formulas and Klainerman's vector field method.

(d) Null condition and small data global existence for nonlinear wave equations satisfying null conditions with slowly decaying initial data, e.g. $\Box\phi = (\partial_t \phi)^2 - |\nabla\phi|^2$.

(e) Failure of global existence without null condition, e.g. $\Box\phi = (\partial_t\phi)^2$.

(f) Basic differential geometry: metric tensor, Levi-Cevita connection, Riemann curvature tensor, Ricci curvature.

(g) Initial value formulation of Einstein vacuum equations.

2 Fluid Mechanics

(a) Local Well Posedness for Euler and Navier Stokes in 3D (Chapters 2 and 3 of [BV22]).

(b) Beal-Kato-Majda Criterion for Euler (Chapter 2 of [BV22]).

(c) Global Well Posedness in 2D for Euler (Chapter 2 of [BV22]).

(d) Mild solutions for Navier Stokes and semigroups (Chapter 5 of [BV22]).

(e) Yudovich theory of vorticity solutions to 2D incompressible Euler (Chapter 8.2 of [MB01]).

3 Harmonic Analysis

(a) Statements of basic interpolation theorems (Marcinkiewicz, Riesz-Thorin), properties and interpolation with weak L^p spaces.

(b) Singular integral operators: Calderon-Zygmund kernels, stopping time decomposition, L^p boundedness, boundedness on Holder spaces (Chapter 7 of [MS13]).

(c) Littlewood–Paley theory: Mikhlin multiplier theorem, Littlewood–Paley square-function estimate (Chapter 8 of [MS13]).

(d) Almost orthogonality: Cotlar's lemma, Schur's lemma, Calderon–Vaillancourt theorem, Hardy's inequality (Chapter 9 of [MS13]).

References

[BV22] Jacob Bedrossian and Vlad Vicol. *The mathematical analysis of the incompressible Euler and Navier-Stokes equations—an introduction*. Vol. 225. Graduate Studies in Mathematics. American Mathematical Society, Providence, RI, 2022, pp. xiii+218. ISBN: [9781470470494]. DOI: 10.1090/gsm/225. URL: https://doi.org/10.1090/gsm/225.

[MB01] Andrew J. Majda and Andrea L. Bertozzi. *Vorticity and Incompressible Flow*. Cambridge Texts in Applied Mathematics. Cambridge University Press, 2001.

[MS13] Camil Muscalu and Wilhelm Schlag. *Classical and Multilinear Harmonic Analysis*. Cambridge Studies in Advanced Mathematics. Cambridge University Press, 2013.